Mathematical Tasks and Learning Goals: Examples from Japanese Lesson Study

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This paper explores how to select, or design, the best mathematical task for a given learning goal. Examples are taken from a recent project in Victorian Primary schools that employed Japanese Lesson Study as the means to provide teachers with Professional Learning within their own classrooms[1]. The discussions by participating teachers and researchers provide some insights into the difficulties and solutions facing teachers intending to improve the critical thinking skills of their pupils. Examples of tasks and goals are provided.

The data for this paper were collected during eight hours of discussions held by Year 3 and 4 teachers from three Victorian Primary schools, from a research lesson, and the post-lesson discussion. These teachers were taking part in a small-scale project investigating the cultural factors aiding, or impeding, the use of Japanese Lesson Study (Janzen, 2005, Takahashi, 2013) in Victorian mathematics classrooms, following earlier work by Groves and Doig (2010) and Doig, Groves, and Machackova (2009) in this area.

The focus of these planning discussions was the preparation of a Research Lesson to be taught with a public audience of observers. The issues facing the participant teachers, one of whom would teach the research lesson on behalf of the planning team, provide an example of how goals and tasks are linked.

What is a task?

Japanese Lesson Study is composed of a series of phases leading to the teaching in public of a Structured Problem Solving Lesson (Doig, Groves, *et al*., 2009; Doig & Groves, 2011), and so the notion of a *task* is inextricably linked with the notion of a *problem*. In Japanese, however, the term used is *hatsumon,* which encompasses both the manner of presenting the task as well as the actual task itself.

Outside Japan, the definition of "task" within the mathematics classroom has a range of meaning as suggested by Watson and Mason (2007) in which they argue that the definition of task is not taken-as-shared in mathematics education. They suggest that a task "in the full sense includes the activity which results from learners embarking on a task" (p. 207), which is somewhat akin to the *hatsumon*. Further, Doig Groves and Fujii (2011) note that what is important is that whatever the term, it is the "stimulus activity" (pg. 182) for the learning of some aspect of mathematics. That is to say, one may use task and problem interchangeably, if their common purpose is to stimulate mathematical learning. Furthermore, the Japanese curriculum in mathematics for primary schools, at least in its English translation, begins an instructional section with the phrase "through mathematical activities" (Isoda, 2010, p. 323), and one may suppose, then, that the distinction between

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the various terms is un-necessary, and so in the discussion that follows, the term "task" will be used to name the *hatsumon*, whether it be a task in the Watson and Mason sense or a problem in the sense of a problem-solving endeavour set for pupils, or simply a mathematical activity.

Learning goals

An example of a *hatsumon,* in a Japanese Structured Problem Solving Lesson, is the presentation to pupils of the following: 13-9 = ? However, to find the answer – 4 – was not the task. The teacher's goal was for the pupils to find a strategy for solving the task, not merely to find the answer: in fact, the teacher asked the pupils if they knew the answer, and all of them did. Then the *real* task was put to the pupils (Doig, Groves, Fujii, 2011). It should be noted that in the Japanese Structured Problem Solving Lesson, only one task is presented to the pupils, and the discussion stimulated by the *hatsumon* takes the major part of the lesson time.

In the second cycle of the Japanese Lesson Study Project (JLSP) teacher discussion initially focussed on what goal would be useful for Year 3 and Year 4 pupils late in the school year (December in Australia) when many teachers are revising the year's content and not introducing new content. A suggestion was made to look at what new content, for Year 3, would be good consolidation for Year 4, and this immediately raised the idea of some content focussed on Place Value.

The hunt for a Place Value task that the Planning Team would find acceptable led to a search of most of the on-line resources provided by the Victorian Department of Education and Early Childhood Development (DEECD) and yet nothing was found that everyone in the team would accept. The university researchers leading the JLSP also searched journals and the Internet, and found a dearth of suitable tasks.

The university researchers and the Planning Team looked at the Australian Curriculum – Mathematics (ACARA, 2010), and the Victorian adaptations known as AusVELS (VCAA, 2012) seeking objectives or content in Place Value that might be appropriate. However, while no specific content was thought entirely appropriate, the Level 3 Number and Algebra strand description: "Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems" (ACARA, 2012) appeared to be a suitable goal. This decision taken, the search then continued for a task that addressed the goal of partitioning of a number to either solve a problem or assist in a calculation. However, as the JLSP participants discovered, most Place Value activities were focussed either upon learning about Place Value or teaching what it is, rather than using it to solve a problem.

Finding tasks

Mason and Johnston-Wilder (2004) argue that the purpose of a mathematical task is "to initiate mathematically fruitful activity" (p.25). Further, Breen and O'Shea (2010) report that Mason and Johnston-Wilder also suggest "tasks should involve a range of possibilities and offer students opportunities to discuss ideas" (p. 43). This view echoes the sentiments of Japanese teachers when they consider task selection for their Research Lessons.

A previous JLSP Research Lesson had used a visual stimulus as part of the *hatsumon*, and the researchers felt that this would be an idea to follow in this Research Lesson too. In fact, a perusal of Japanese pupil mathematics textbooks, showed a high level of visual stimuli as part of the *hatsumon*. In most instances the visual acted as a means of

representing the mathematics to be studied and thus was meant to be used by the pupils as the basis for their solutions.

The search for a task that was appropriate in both the curriculum content and Research Lesson senses continued. Eventually, Project leaders found a suitable multiplication lesson that retained elements of the previous Place Value goal. The task visual stimulus is shown in Figure 1 and the question to be put to the pupils was "Can you find a way to work out the total number of dots without counting them all one-by-one?":

Figure 1: Visual stimulus for the selected Research Lesson task.

This is essentially 23 x 3=? a first step towards a vertical multiplication algorithm. (This task can be found on the web-site: http://homepages.warwick.ac.uk/staff/David.Tall).

JLSP teachers initially queried the use of 23 and 3, asking why this was the question rather than, say, 24 x 3. Discussion continued until the Planning Team agreed that 23 x 3 was a large enough product to help pupils see the need for an efficient strategy, and that the complication of having a sub-product with a ten (such as in 23 x 4 = 80+12) was avoided. Satisfied now that these particular numbers were the best choice, and that the visual stimulus was appropriate, the Planning Team then set about finding a way to put the task to the pupils. A key factor in this discussion was that some of the Year 4 students would simply write a number sentence or vertical algorithm and write the answer. The solution was to ask all pupils to use the diagram to find a strategy for solution, thereby ensuring that those who used an algorithm would still need to show their understanding of their algorithm. Members of the Planning Team, who were not going to teach the Research Lesson, trialled different wordings to help decide on the final wording for the Research Lesson. A final version used was: "Here are some dots on a page. Use the diagram to show your thinking to work out how many dots there are altogether".

The Outside Expert commentator, Professor Fujii from Tokyo Gakugei University, asked why did the JLSP use 23 by 3 rather than other numbers. He was told the reasons cited above, and he then went on to point out that 23 is a prime number and therefore, unlike 24, cannot be broken into factors, that introduces the possible solution of 24 x 3 = (6 x 4 x 3). Another reason is that 23 is more likely to be seen by pupils as 20 and 3, or 10, 10, and 3, for which pupils already know their products of 3. Thus, expected solutions included 10 x 3+10 x 3+3 x 3 through partitioning the set of dots at Place Values.

This deep thinking about the details of the elements of a task is part of the *kyozaikenkyu,* in Japanese, which examines curriculum and resources, and is a thorough investigation of the final task. The *kyozaikenkyu* is like an iceberg, where a large amount of support for the smaller visible feature, is unseen but essential.

Conclusion

Japanese Lesson Study advocates and users are not the only ones who generate good tasks for mathematics learning. Doig, Groves, and Fujii (2011) give an example from a Victorian Preparatory Year teacher: *Snow White and the Seven Dwarfs* (p. 187). In this example, the teacher wanted her pupils to find many solutions and know when they had them all. The task started with the pupils finding, by drawing or using concrete materials,

different ways the seven dwarfs could be seated along the sides of their kitchen table. After some exploration of this, the teacher then asked her pupils "Do we have all the ways?" After some discussion, she then posed the real task: "How do you know if we have found all the ways?" Much discussion was had before a pupil proposed that "We have used up all the numbers" meaning 0 and 7, 1 and 6, etc., up to 7 and 0.

This paper has described part of the journey of one Planning Team as it sought to prepare a Research Lesson for their Japanese Lesson Study Professional Development experience. It is important to note what was learned in the process of planning the lesson was that: Curriculum resources need to be carefully examined, particularly for goals; suitable tasks are hard to find, but they do exist; and that trialling different ways of posing the task is very instructive to those planning a Research Lesson.

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